Problem 4

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+1)^2}$$

Solution

Take the limit of the summand as $n \to \infty$.

$$\lim_{n\to\infty} \frac{(-1)^n n^2}{(n+1)^2} = \left[\lim_{n\to\infty} (-1)^n\right] \left[\lim_{n\to\infty} \frac{n^2}{(n+1)^2}\right]$$

$$= \left[\lim_{n\to\infty} (-1)^n\right] \left(\lim_{n\to\infty} \frac{n^2}{n^2 + 2n + 1}\right)$$

$$= \left[\lim_{n\to\infty} (-1)^n\right] \left(\lim_{n\to\infty} \frac{1}{1 + \frac{2}{n} + \frac{1}{n^2}}\right)$$

$$= \left[\lim_{n\to\infty} (-1)^n\right] \left(\frac{1}{1 + 0 + 0}\right)$$

$$= \lim_{n\to\infty} (-1)^n$$

$$= \text{does not exist}$$

Since it's not zero, the series diverges by the preliminary test.